
UNIT 4 LEVELLING

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4.1 INTRODUCTION

The purpose of levelling is to determine the relative heights of points of interest on the surface of earth. Thus, it involves measurements of distances in a vertical plane like distances in horizontal planes were measured in chain surveying. The levelling exercise will provide an accurate network of elevations of ground surface covering the entire area of the project site. For many civil engineering projects, the levelling is of critical importance. For construction of highways, canals, pipelines of water, gas or sewage, railway tracks, dams etc., the accurate knowledge of relative heights of ground surface along its alignment and cross sectional details at suitable intervals is essential for their execution. **The basic design and project economy of such projects depend on the accuracy with which the levelling is carried out.** A good network of levels provides an excellent idea of the existing terrain in terms of relative heights, based on which an engineer can plan, design and execute the project safely, effectively and economically.

Objectives

After studying this unit, you should be able to

- understand the basic principles of levelling and various terms used in levelling,
- explain the use and working of different types of levels and other instruments used in levelling,
- explain various operations and procedures performed during levelling exercise, and
- describe various methods of contouring and uses of contour maps.

4.2 DEFINITIONS OF BASIC TERMS

Level Surface

Any level surface parallel to mean spherical surface of earth is called a level surface. This surface is normal to the direction of gravity (indicated by plumb bob). Every point on this surface is equidistant from centre of earth. A plane tangential to level surface is called the horizontal plane at that point. Any line lying in the horizontal plane is a horizontal line as shown in Figure 4.1(a).

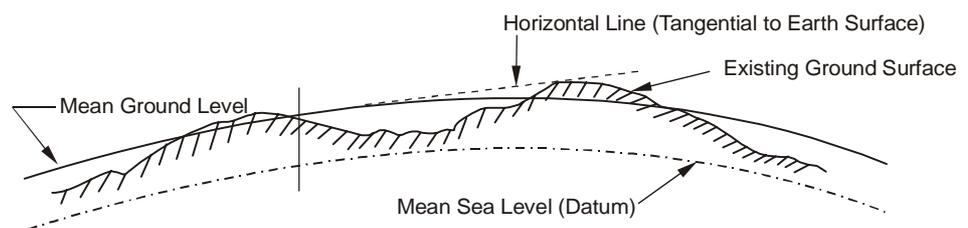


Figure 4.1(a) : Level Surface

Vertical Plane

The plane normal to horizontal plane at any point will be the vertical plane. This plane will contain the plumb line drawn through that point. The angle of intersection between two lines in a vertical plane is called vertical angle. It is normal to select horizontal line as one of these two lines to measure the vertical angle (Figure 4.1(b)).

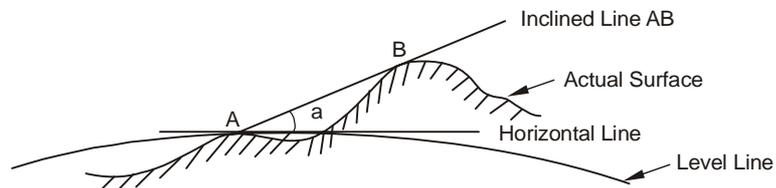


Figure 4.1(b) : Vertical Angle

Datum

Since the actual ground surface of earth is undulating, one reference line has to be decided to obtain the relative heights of points on ground on the surface of earth in the plot of area surveyed. This arbitrarily decided level surface is called datum surface. The heights of different points in surveyed area are measured with reference to this level surface. In India the datum was fixed as mean sea level at Karachi during Great Triangulation Survey

(GTS). This datum is still being used for benchmarking in all precision surveys.

Bench Mark

A bench mark (BM) is a fixed (permanent) reference point of known elevation. This bench mark is used as a base relative to which the elevations or levels of different points in a survey are measured. This could be

- (a) GTS bench marks
- (b) Permanent bench marks
- (c) Temporary bench marks
- (d) Arbitrary bench marks

GTS bench marks are established precisely and accurately by Survey of India department and are used as base for all levelling exercises, particularly when large areas are to be surveyed. Reference bench marks fixed in an area on permanent structure are called **permanent benchmarks**. These are used for reference and future surveys to provide continuity. In small levelling works, the reduced level of a well defined reference point is assumed as **arbitrary benchmark** of levels. During the levelling exercise, whenever there is a break of work continuity, **temporary bench marks** are established to provide continuity when the survey is resumed.

Reduced Level

The elevation of a point is its vertical distance above or below the datum line. This is also known as the reduced level (RL) of the point.

Line of Collimation

It is the line joining the point of intersection of cross hairs to the optical centre of object glass. It is also called the line of sight. This indicates the horizontal line at the station of instrument at an elevation of instrument height. While the axis of instrument will be the line joining the optical centre of the object glass to centre of eyepiece, height of instrument is the level of the plane of collimation.

Backsight and Foresight (BS and FS)

The staff reading taken at a point of known or predetermined elevation, e.g. a bench mark is termed backsight or plus (+) sight. It is the first staff reading taken after setting the instrument at specified survey station. The foresight is the staff reading of the point whose elevation is required to be obtained, particularly at a change point. It is the last staff reading at the station before the instrument is shifted to a new station. All other staff readings taken at different points of interest of unknown elevations from one instrumental set up between the back sight and fore sight are called intermediate sights (IS).

Station

Any point on ground whose level is required to be determined or a point whose level is already fixed (e.g. a bench mark) is termed as a station. It is to be noted that it is a point at which level measuring staff is positioned and not the point at which the instrument is setup.

A Turning Point (TP) or a Change Point (CP)

A turning point or change point denotes the position at which both foresight and backsight readings are taken before shifting of level instrument. Any

well defined and stable point can be selected as change point, e.g. boundary stone, benchmark.

SAQ 1 

Define the following terms :

- (a) line of collimation,
- (b) bench mark,
- (c) change point, and
- (d) backsight and foresight.

4.3 BASIC PRINCIPLE

A levelling exercise is required when difference between levels of two points is to be determined. This could be simple levelling, when both the points are visible from a single instrument setting (Figure 4.2(a)) or differential levelling (Figure 4.2(b)) when the two points are quite far apart, the level difference is large or there are obstacles in the line of sight.

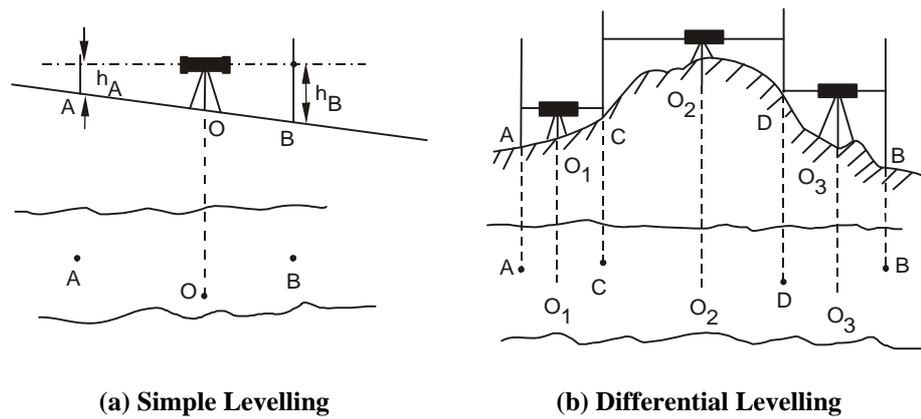


Figure 4.2

In simple levelling, instrument is set approximately at midway (at *O*) between stations *A* and *B*. Staff is held at *A* then at *B* and corresponding levels taken and recorded. Let these be r_a and r_b respectively. The difference between r_a and r_b gives the level difference between stations *A* and *B*. It may be noted that when station is lower the staff reading is greater and vice-versa. The instrument's telescope must remain horizontal while taking the readings. This implies that bubble of the spirit level attached to instrument must be kept to mid position during the entire levelling exercise at position of instrument setup. If the levelling of instrument is required during observation period, it will indicate that some error is introduced in the measurement process.

4.4 INSTRUMENTS AND EQUIPMENT

4.4.1 Different Types of Level

To determine the ground level at any point, two equipment, i.e. a level and a leveling staff are needed. A horizontal line of sight with a cross wire is provided

by the level while the levelling staff provides the vertical distance of ground station from the line of sight.

Level

Various types of levelling instruments used can be listed as follows

- (a) Dumpy level
- (b) The Wye or Y level
- (c) Cooke’s reversible level
- (d) Cushing’s level
- (e) Tilting level and
- (f) Automatic level

Dumpy Level is by far the most commonly used level in engineering surveys and hence described in detail here. Only important characteristics of other types of levels are mentioned.

Dumpy level is a simple, stable and compact instrument with several components as shown in Figure 4.3.

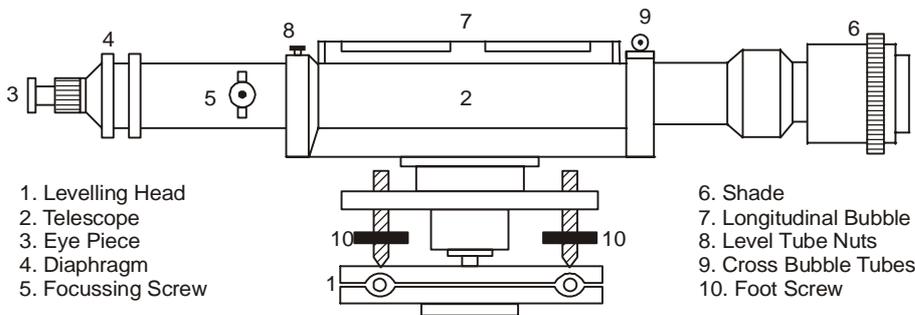


Figure 4.3 : Dumpy Level

The telescope is rigidly attached to supports with a longitudinal bubble tube fixed at its top. The telescopic tube cannot be rotated about its horizontal axis, nor it can be removed from the supports. A transverse bubble tube at right angles to main bubble tube is provided to adjust the level’s plane in horizontal position. The telescope consists of object glass, eye piece and a circular ring diaphragm with cross wires. The telescope usually has a magnification factor of thirty. The eye piece can be rotated to bring crosswire of the diaphragm distinct and clear. Focusing screw helps in bringing the object’s unique image into the diaphragm plane.

The levelling head is provided with two rigid parallel plates with three screws to level the instrument. In some instruments, a compass is also provided below the telescope to measure bearings. Since the dumpy level has only a few movable parts, it is simply to be set and adjusted which are not easily disturbed providing sturdiness to the instrument.

Y levels are sensitive and delicate instruments. The telescope is removable and can be reversed end to end. This provides the facility of making indoor adjustment easily and rapidly.

In **Cushing’s level**, though the telescope can’t be removed, the eye piece along with diaphragm and the object glass are removable and hence can be interchanged.

The **Tilting levels** are used for precision levelling. The telescope can have a small motion about a horizontal axis just below the telescope axis.

Automatic levels are also termed as self aligning levels. The levelling of instrument is automatic as against manual levelling by spirit bubbles in conventional levels.

4.4.2 Levelling Staff

It is a device to measure the distance by which the staff station is above or below the line of sight. It could be made of well seasoned timber or aluminum, generally having size of 75 mm × 25 mm, with graduations marked, starting from foot of staff as zero and increasing upwards. The commonly used staff (Telescopic staff) is usually 4 meters long with specifications as laid in IS : 1779-1961. It has three telescopic lengths. Top solid piece is about 1.2 meter long slides into the central box of about 1.3 m length. The lower base is of 1.5 m length as shown in Figure 4.4. The inner pieces can be pulled out and latched by metal spring clamps to remain stable. The graduation

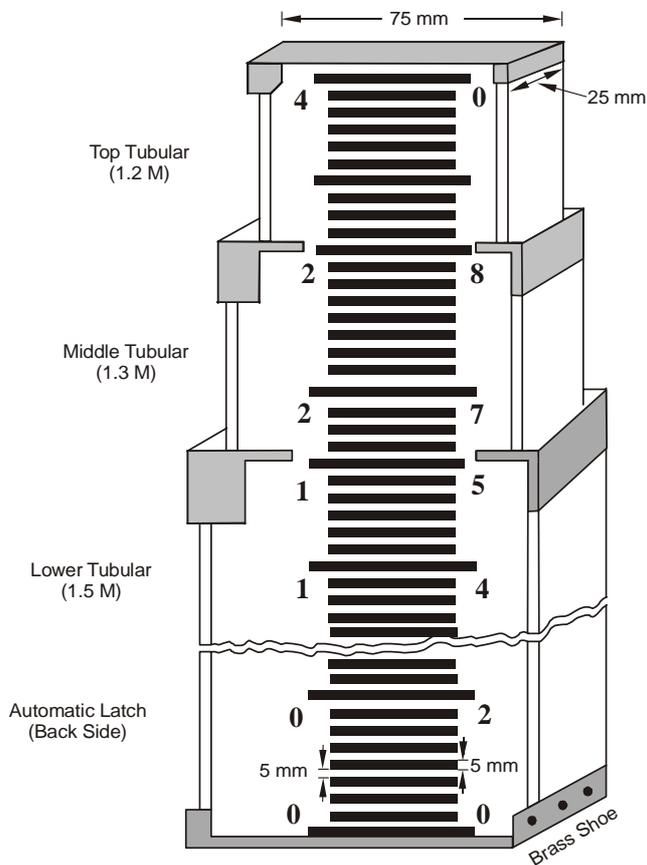


Figure 4.4 : Telescopic Levelling Staff

markings are painted black against a white background. The 4 m long folding staff usually of aluminum are also available consisting of two 2 m long pieces with hinged joint at middle, along with the locking device. Each meter is divided into 200 divisions with each graduation 5 mm thick as shown. **Invar precision levelling staff** may be used when high precision measurements are needed. Invar graduated band is detachable and is fitted tightly on the wooden staff at lower end and held in position at upper end by a spring. The staff is set on an iron base plate and kept vertical by detachable stays. The reading is obtained with the help of a parallel plate micrometer of the telescope.

4.4.3 Adjustment of Dumpy Level

The level measurement exercise is initiated with taking out the instrument from the box and making two types of adjustments, i.e. temporary and permanent adjustments, before any measurements are taken.

The positions of the object glass, the eye piece and the clamp etc. are carefully marked before taking out the instrument from the box. This ensures that the instrument can be placed back in the box without any difficulty and damage in its proper position at the end of work day.

Temporary Adjustments

Temporary adjustments are required to be carried out at each set up of level before taking any reading. Usually following step by step exercise is recommended.

Setting Up

The tripod legs are properly spread on the ground and the clamp screw of level is released. The level is taken in the right hand and is fixed on the tripod by turning round the lower part with the left hand. The tripod is then placed on ground in the desired position so that the level is at a convenient height for sighting. The foot screws are initially turned so that they are in the centre of their run while the legs of tripod are spread such that the level is as nearly horizontal as possibly judged by eye.

Two of the legs are firmly pressed in the ground and the third leg is moved to the left or right until the transverse level bubble is approximately in the middle. The telescope is brought parallel to the firmly pressed legs of tripod (i.e. the pair of foot screws) and the bubble of main level tube is brought into exact middle position by turning these foot screws either inward or outward simultaneously. The telescope is then turned by 90° so that it lies over the third foot screw. The bubble is centered again by turning this third screw only. Rotate the telescope by 90° once again clockwise and check the bubble. If there is any displacement, bring back the bubble in its central position. This cycle of exercise is repeated till the bubble traverses such that it remains in central position for all positions of telescope. In this position, the instrument is considered to be properly positioned and leveled.

If the telescope is turned through 180° and the bubble position from middle is displaced, it indicates that the instrument requires to be repaired and corrected by making permanent adjustments.

After levelling the instrument, its focusing is required. The eye piece and object glass are focused consecutively. The lid is removed from the object glass, and a piece of white paper is held before it. The eye piece is moved in or out till the cross-hairs are clearly and distinctly seen. The telescope is then rotated towards the staff. The image of staff is brought between the two vertical hairs of the diaphragm by use of a tangent screw. The focusing screw is then adjusted until the parallax between direct sight of staff and its image is removed. The instrument is now ready for making the measurements, i.e. the line of collimation is now perfectly horizontal.

Permanent Adjustments

The three critical axes of a dumpy level are line of collimation, bubble tube axis and instruments vertical axis as shown in Figure 4.5. These are correlated in following way with each other.

- (a) Line of collimation and bubble tube axis are parallel to each other.
- (b) The bubble tube axis is normal to vertical axis of the instrument.

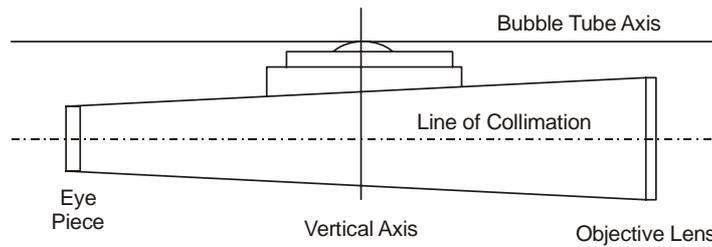


Figure 4.5 : Critical Axes of Dumpy Level

These conditions are ensured by the manufacturer during production of instrument. However, due to continuous usage some wear and tear do occur and the above relationships between critical axes are disturbed. This will introduce instrumental error in level measurements. For accuracy, the level is required to be adjusted so as to satisfy the above conditions. Since these adjustments are needed only when the instrument’s internal setting is disturbed during usage and not required every time the instrument is setup, these are termed permanent adjustments.

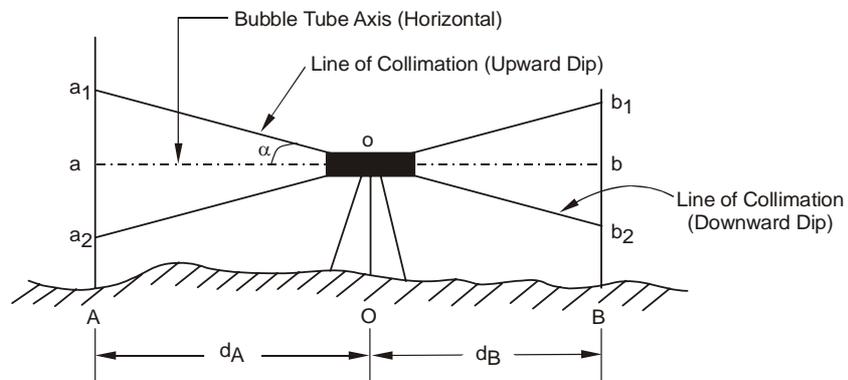


Figure 4.6 : Level out of Adjustment

When the instrument is not adjusted, the line of collimation will not be parallel to the bubble tube axis (horizontal) but inclined either upwards or downwards (Figure 4.6). Let this inclination be α . An error is introduced in recording the levels. Let us determine the true difference in levels of station A and B situated at a distance of d_A and d_B respectively from instrument setup at O. Then line aob represents the true line of collimation (horizontal). However, since the adjustments are out of order, the actual line of collimation will be a_1ob_1 with downward dip and a_2ob_2 with upward dip of line of collimation.

If Aa_1 = observed reading of staff at station A

Then error in reading at A will be $Aa_1 - Aa = d_A \tan \alpha$

Similarly, when staff is held at station *B*,

Bb_1 = Observed reading; Bb = actual reading

Error in reading at *B* is $Bb_1 - Bb = d_B \tan \alpha$.

True difference in levels of stations *A* and *B* will be

$$Bb - Aa = \{Bb_1 - d_B \tan \alpha\} - \{Aa_1 - d_A \tan \alpha\}$$

$$= \{Bb_1 - Aa_1\} - \{d_B \tan \alpha - d_A \tan \alpha\} \dots (4.1a)$$

Eq. (4.1a) will indicate the error in adjustment and also indicate that it is proportional to distance of staff station from instrument station. One way to eliminate this error would be to keep the instrument station *O* at exactly equidistant from staff stations *A* and *B*, i.e. $d_A = d_B$. It will modify Eq. (4.1a) as

$$[Bb - Aa] = \text{True level difference at A and B}$$

$$= [Bb_1 - Aa_1] \dots (4.1b)$$

4.5 METHODS OF LEVELLING

4.5.1 Step-by-step Procedure

When the instrument is set in position, leveled and adjusted, the line of collimation will be horizontal and the bubble will traverse, i.e. it will remain central when the telescope is rotated in the horizontal plane of collimation. The instrument is ready to record vertical measurements by reading the measuring staff positioned at required stations.

While holding the staff in position at station, it should be held vertical. The survey helper is asked to stand behind the staff and hold it between the palms. The helper will tilt the staff forward and backward so that the surveyor will record the minimum staff reading to indicate true vertical position (Figure 4.7). In precision levelling, the measuring staff is provided with a circular bubble at the back to hold it in plumb. The staff is brought in the line of sight between two vertical lines/hairs etched on the diaphragm and the reading is recorded where horizontal hair of crosswire cuts the staff.

First of the two steps in finding the level at a station is to find the elevation or reduced level (*RL*) of the plane of collimation, also known as height of instrument (*HI*). This is achieved by taking a back sight on a bench mark once the instrument is set in position. The second step is to find the elevation or *RL* of all other desired stations, e.g. intermediate and change points (Figure 4.7).

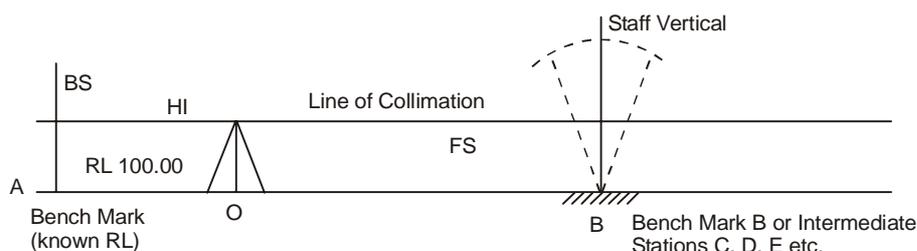


Figure 4.7 : True Vertical Position of Staff

Height of Instrument, $HI = RL$ of the line of collimation

$$= RL \text{ of bench mark} + \text{Backsight on } BM$$

$$\begin{aligned}
 RL \text{ of any desired station } (B) &= HI - \text{Foresight at } B \\
 &= HI - \text{Intermediate sight at } B \text{ (If } B \text{ is an} \\
 &\quad \text{intermediate station)}.
 \end{aligned}$$

4.5.2 Simple and Differential Levelling

As mentioned earlier, simple levelling is the simplest levelling operation (Figure 4.2(a)). Let the staff readings at A and B stations be H_A and H_B . If RL of A is 100.00, RL of station B can be obtained as follows

$$\begin{aligned}
 HI \text{ at } O &= 100.00 + H_A \\
 RL \text{ of } B &= 100.00 + H_A - H_B
 \end{aligned}$$

And the level difference between A and $B = [H_A - H_B]$.

The case of differential levelling is explained earlier and shown in Figure 4.2(b). The instrument is set and leveled at O_1 and staff reading of station A (Backsight) of known level (say a bench mark) is taken. Station C in a firm ground (change point) is selected and staff reading at C recorded from O_1 (foresight). Stations A and C are visible from instrument station O_1 . O_1C is approximately taken as O_1A to minimize error due to line of collimation not being exactly horizontal.

Instrument can then be shifted to another station O_2 from which change point C and another selected change point D (such that $O_2C = O_2D$) are visible. Staff reading, by instrument at O_2 , of station C (backsight) and of station D (foresight) are recorded. Several change points, i.e. E, F, G etc. can be chosen along with instrument stations O_3, O_4, O_5 etc. till last change point and station B are visible from last instrument station. The staff reading of last change point from last instrument station will be a backsight, while that at station B will be foresight. The reduced level (RL) of station B can be obtained from a series of calculations if RL of A is known. Hence level difference between A and B can be obtained. Let the RL of A is RL_A , then

$$\begin{aligned}
 (HI)_A &= RL_A + h_A \text{ (BS at } A) \\
 RL \text{ of } C &= RL_A + h_A - h_C \text{ (FS at } C) = RL + BS_A - FS_C \\
 RL \text{ of } D &= RL_A + BS_A - FS_C + BS_C - FS_D \\
 &= RL_A + (BS_A + BS_C) - (FS_C + FS_D) \\
 RL \text{ of } B &= RL_A + \sum BS - \sum FS
 \end{aligned}$$

Hence, RL of $B = RL_A +$ Difference between sum of all backsights and sum of all foresights.

And level difference between A and $B =$ Difference between sum of backsights and sum of foresights.

4.5.3 Reduction of Levels

In previous section, a method of computing reduced levels and hence the level difference of various stations of interest was explained. In general, there are two basic methods of computing elevation of RL of points from field levelling exercise which are

- (a) the collimation system or instrument height system, and
- (b) the rise and fall method.

Collimation Method

As explained earlier, the height of instrument (*HI*), e.g. the height of line of collimation above *BM* (station of known level) at each instrument station is determined by adding the backsight of *BM* station to reduced level of *BM*. From this height of instrument at a particular instrument station, reduced levels of all the station points on ground are calculated by subtracting foresight of that particular station from *HI*, i.e.

$$HI \text{ of instrument} = RL \text{ of Bench mark} + BS \text{ of } BM$$

$$RL \text{ of intermediate point} = HI - FS \text{ at intermediate station} \\ = HI - IS$$

When the instrument is shifted to its second position, height of instrument at new set up station is required to be determined. This is achieved by correlating the levels of two collimation planes (first and second position) by foresight of change point from first setup station and backsight of same change point from second setup station, as follows :

$$RL \text{ of change point } C = RL \text{ of } A + BS \text{ at } A - FS \text{ at } C$$

$$HI \text{ (at second station } O_2) = RL \text{ of } C + BS \text{ at } C$$

With instrument set up at second station (say *O₂*), staff readings at new system of intermediate stations are taken before shifting the instrument at next set up station (*O₃*). This process is continuously repeated till the levelling exercise is completed, and all the required reduced levels are obtained.

A check can be applied on the mathematical correctness of calculation of reduced levels by collimation method as follows. The difference between the first reduced level (at starting station) and last reduced level (at end station) must be equal to the difference between summation of all foresights at change points and the summation of all backsights at change points.

The Rise and Fall Method

Instead of finding the instrument height at a setup station, the difference between consecutive points is obtained from their staff readings with that immediately preceding it. The difference indicates a rise or a fall. The reduced level of each point is then obtained by adding the rise to or subtracting the fall from the *RL* of the preceding point. The arithmetic check in this method is as follows :

$$\sum BS - \sum FS = \sum \text{Rise} - \sum \text{Fall} \\ = \text{Last } RL - \text{First } RL$$

It can be noted that the first method of collimation is simpler and faster than the rise and fall method. However, there is no check in reduction of levels at intermediate stations in collimation method while the second method provides arithmetic check on all the level reductions. We can conclude that the collimation method can be preferred for profile levelling or setting out construction levels, while rise and fall method is preferred for differential levelling, check levelling and other important applications.

Some precautions in recording the measurements in field books should be taken to avoid error in recording and subsequent computations. Care should be taken to make entries strictly in the respective columns prescribed for them in order of their observation. The first entry on a fresh page in field book shall always be a backsight while the last entry is a foresight. If the

last entry happens to be a staff position at intermediate point, instead of a change point, it shall be made both in foresight and backsight columns at the end of the preceding page and as the first entry into the succeeding page. In the remark column, bench marks, change points and other important information shall be briefly but accurately recorded, preferably explained with the help of sketches by free hand drawn on the left side of the page.

4.5.4 Classification of Levelling

The levelling exercise can be classified into several categories depending upon its purpose and applications. Some of the important ones can be listed as follows.

Differential Levelling

When there are obstructions in the line of sight, the distance between stations is too large or the purpose is to establish bench marks, this process is adopted. This is also termed as “fly levelling”.

Check Levelling

It is normal to run a line of levels to return to start station after the end of each days work for the purpose of checking the accuracy and reliability of the measurements and recording carried out on that particular day. This is termed check levelling. This is also carried out to check the particular set of levels fixed previously, or to validate their accuracy.

Reciprocal Levelling

The level differences between two stations are required to be obtained accurately by two independent set of observations particularly when instrument station cannot be set at equidistant position from these stations. The process adopted is known as reciprocal levelling. This process eliminates the errors due to curvature, refraction and collimation. It is described in greater detail in Section 4.5.5.

Trigonometric Levelling

When the level differences between stations is very large, e.g. valleys and mountains, the levelling process becomes too tedious and complex, even impossible in some cases. It is much simpler and faster to measure level differences by measuring vertical angles by theodolite and horizontal distances either by Chaining or Tacheometry. The process is called trigonometric levelling because trigonometric relations are used in computations.

Barometric/Hypsometry Levelling

The altitudes or vertical elevations of objects, e.g. mountains, are obtained by measuring atmospheric pressures by use of barometer. It is based on the fact that atmospheric pressure at a point depends on the elevation above mean sea level (MSL) reducing gradually with the height. When measurement of temperature at boiling point of water is used to obtain the height of station above MSL, the method is called hypsometry.

Profile Levelling

When levelling exercise is undertaken along a survey line, e.g. deciding the route of a road or railway line, centre line of a pipe/gasline, power/telephone lines etc., it is termed as profile levelling. The levelling exercise along the survey line is termed longitudinal levelling while cross

sectional levelling is conducted to determine surface undulations transverse to longitudinal levelling line. The profile levelling is described in detail in Section 4.5.6

Contouring

When the elevations and depressions of various points on the ground are required over widespread areas rather than along a width of area, the levelling exercise known as contouring is resorted to. A line joining the points of same level (height or RL) is called a contour line representing the horizontal section of the ground surface at that elevation. The area to be surveyed is generally divided by a square grid of predetermined size or a system centre points and radial lines and actually plotted on the ground. The RLs of points of intersections are then obtained by usual levelling methods. The contour lines at predetermined intervals are then interpolated and drawn on a map. This map representing a three dimensional information of the ground is called contour plan. This is discussed in detail in Section 4.5.7.

4.5.5 Reciprocal Levelling

Let the level difference between stations *A* and *B* is required to be measured precisely, and it is not possible to set up instrument midway between *A* and *B* (Figure 4.8). At first, instrument is set up and levelled very near to *A* and staff is held at *A* and *B* and readings recorded respectively. The instrument is then shifted to a position *O*₂ very near to station *B*, and staff readings once again taken for station *A* and *B* respectively.

Let *a*₁ and *b*₁ be the readings from position *O*₁, while *a*₂ and *b*₂ are readings with instrument at *O*₂. If there is no collimation error, the line of collimation should coincide with horizontal line. If there is no error due to curvature and refraction etc., the level line should merge with horizontal line. Thus, in an error free environment there shall be a single line in place of three, both in instrument position at *O*₁ (Figure 4.8(a)) and at *O*₂ (Figure 4.8(b)).

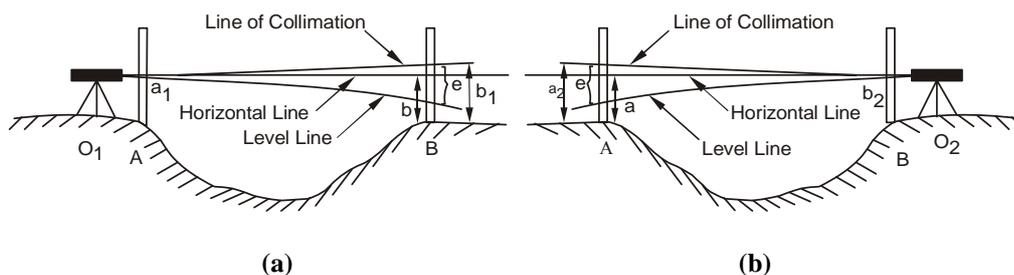


Figure 4.8 : Reciprocal Levelling

Since instrument position *O*₁ is very near to or exactly at *A*, there will be practically no deviation in staff reading at *A* (i.e. reading *a*₁ is correct) while the true reading (level difference between *A* and *B*) would be

$$d = (b_1 - e) - a_1 \quad \dots (4.2)$$

Similarly for instrument position very near to or exactly at *B*, the true level difference would be

$$d = b_2 - a_2 + e \quad \dots (4.3)$$

Adding Eqs. (4.2) and (4.3), we get

$$2d = (b_1 - a_1) + (b_2 - a_2) \text{ or } d = 1/2 \{(b_1 - a_1) + (b_2 - a_2)\} \quad \dots (4.4)$$

Eq. (4.4) does not have a “*e*” term hence is error free. The magnitude of error *e* is obtained by subtracting Eq. (4.3) from Eq. (4.2).

i.e.
$$e = 1/2 \{(b_1 - a_1) - (b_2 - a_2)\}$$

This process of reciprocal levelling eliminates errors due to collimation and curvature. The refraction error may not be completely eliminated as there is a possibility that refraction of the air may change during shifting of instrument from *O*₁ to *O*₂. Hence for more accurate results and to eliminate any possible refraction error, two independent instruments are set at stations *O*₁ and *O*₂ simultaneously and readings *a*₁, *a*₂, *b*₁ and *b*₂ recorded at the same instant.

4.5.6 Profile Levelling

As explained earlier, the ground undulations along a predetermined line (route) for a centre line of road, railway, pipeline, canal or transmission line etc., longitudinal levelling is conducted. Details of ground transverse to longitudinal line at suitable intervals are measured by cross section levelling.

Longitudinal Profile Levelling

Let the central line of required route be *ABCD* as shown in Figure 4.9.

Note that the change points *A, B, C, D* etc are about 30 m to 70 m apart, not more than 100 m in normal conditions. Staff intermediate stations, e.g. 1, 2, . . . 11 etc. are usually 5 to 20 m distant.

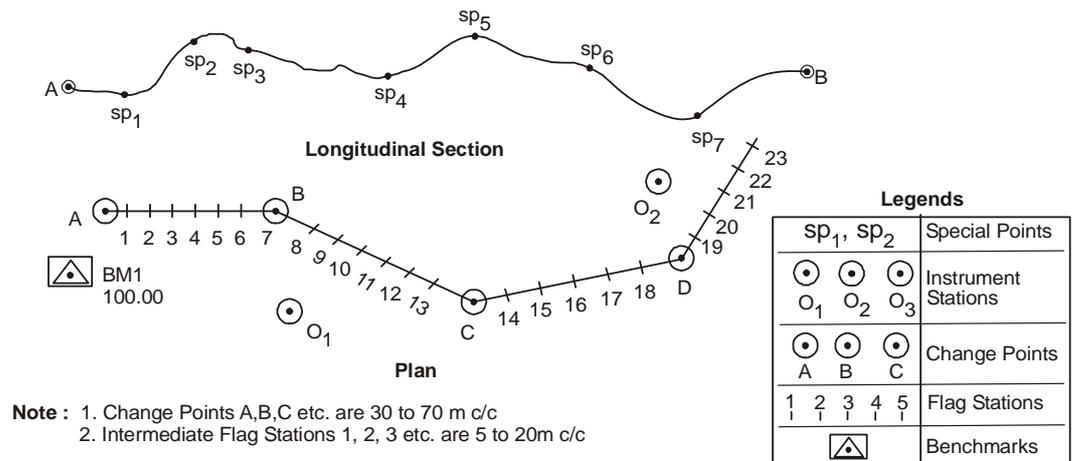


Figure 4.9 : Longitudinal Profile Levelling

The instrument is set up at a suitable firm ground (say *O*₁), properly levelled and adjusted, from which large number of staff stations can be commanded. Backsight is then taken on the bench mark to determine *HI*, the reduced level of line of collimation at instrument station *O*₁.

Staff readings are then taken starting from station *A* followed by readings at predetermined intervals of 5 m or 10 m, measuring the distance *A*-1, 1-2 etc. by stretching the chain on aligned line *AB*. In addition to intermediate stations 1, 2, 3 etc., readings are also taken at critical or important points on the ground, i.e. points indicating change of slope or other important features (e.g. sp 1, sp 2 etc.).

When the length of line of sight exceeds visibility limit, e.g. about 100 m or so, or if there is some obstruction in the line of sight, the instrument is required to be shifted to new position (say *O*₂). Foresight on staff station *B*

is taken from instrument station O_1 before shifting the instrument from position O_1 to O_2 . When the instrument is set, levelled and adjusted at O_2 , the first reading recorded from O_2 will be the backsight at B . This will decide the RL of newly established collimation plane. The distance of intermediate and special points are continued to be measured along line BC and levels read at each of these stations. Previously established benchmarks are important points on which staff readings are necessarily taken as a check on level measuring process. Bench marks can also be used as change points.

To plot the longitudinal profile of the ground along the survey line, first step would be to fix a datum line and marking the chainages of the intermediate, special and change points on it at a suitable scale (Figure 4.10). Vertical lines are then drawn on this chainage line at each intermediate, special and change points. The respective levels are then marked on these lines. The line joining these plotted points represents the longitudinal ground profile. In order to highlight the ground undulations, vertical scale is chosen different than horizontal scale. Normally, the ratio of vertical and horizontal scales is about 10. It can be observed in Figure 4.10 that chainage and levels at each station are written against the ordinates at stations.

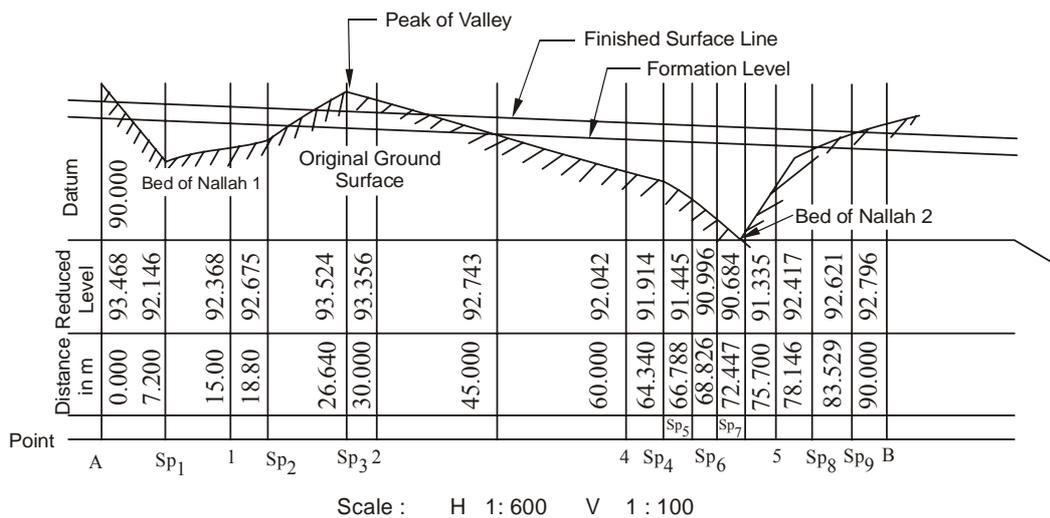


Figure 4.10 : Typical Longitudinal Profile

Based on the longitudinal and cross sectional profiles and the requirements of the project facility, a working profile is prepared. The working profile of a highway is shown in Figure 4.10. It is represented by double line; the lower line indicates the formation level while the upper line will give the finished surface profile. The map will then be able to supply information about

- (a) original ground level,
- (b) formation level,
- (c) finished surface level,
- (d) depth of cutting or filling,
- (e) proposed gradient, and

- (f) any other useful information needed for execution of the construction project.

Cross Sectional Profiling

The project facility whether it is highway, railway, pipeline, or transmission line, will have certain width. Hence, in addition to obtaining information along the longitudinal section, it is also necessary to gather useful information up to desired transverse distance on both side of the line along its entire length.

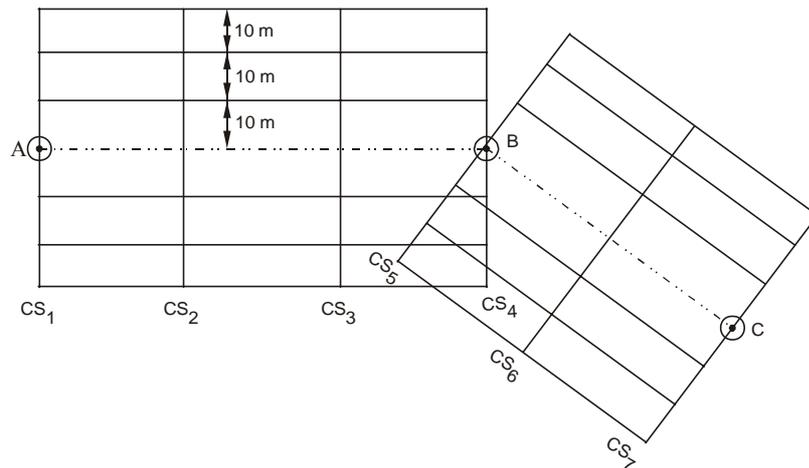


Figure 4.11 : Cross Section Profile

This is achieved by drawing perpendicular lines at desired interval (e.g. 20 m to 30 m) all along the route length. The transverse width (length of cross section) on either side will depend upon the facility requirements. It is 30 m to 60 m for highways, and 200 m to 300 m for railways on each side of the centre line (Figure 4.11). The cross sections are then serially numbered, e.g. CS1, CS2 etc. Along each cross section line, staff intermediate and special stations are determined at which level readings are taken and recorded. The intermediate stations can be at an interval of 10 meters while special stations are fixed at all important points, e.g. points of sudden change of levels.

The recording of readings and drawing the profile is exactly similar to that of longitudinal profiling.

4.5.7 Errors in Levelling

The sources of errors in levelling exercise can be several depending upon the location, instrument employed and human resource. The major sources can be listed as follows

- (a) Instrumental errors.
- (b) Human errors in setting.
- (c) Natural causes.

Instrumental Errors

- (a) The focusing tube may be faulty causing some tilting in line of sight while focusing.
- (b) The bubble may be sluggish or insensitive. It can remain in central position even when bubble tube is not horizontal.

- (c) More common and serious instrumental error is maladjustment of level. The bubble tube line and collimation line do not remain parallel. Even when the bubble tube is horizontal, the collimation line may remain inclined.
- (d) The staff graduations may not be accurate giving wrong results.

Human Errors

Inaccurate levelling of instrument by surveyor while setting the instrument, or settling of level during surveying introduces errors. The error is cumulative. The error can be avoided by taking care to set the level in a firm ground and levelling it carefully. If setting on soft ground cannot be avoided, the legs of level tripod are kept on wooden platform or on stakes driven in the ground. The same precaution can be taken at change and intermediate stations to avoid staff settlement. Care should be taken to avoid any contact with tripod while sighting and taking the staff reading. Other human errors could be error in focusing or staff not being held perfectly vertical while taking the level readings, wrong recording of readings or recording in wrong columns etc.

Natural Causes

These are effects of wind and sun. Considerable difficulty could be experienced while taking the staff reading under glaring sun, or sun shining on the objective glass. Accuracy of observation can also be affected when the velocity of wind is large or when the atmosphere is heated. When the sights are long during precision levelling the errors due to effect of curvature and refraction shall be taken into account. The line of level, defined as a line of equal altitude, will not remain horizontal in long sights due to earth's curvature (Figure 4.12). Aa' will be the recorded level at A while the real level should be Aa . Thus, an error $e = aa'$ is introduced due to earth's curvature given as $e_c = 0.0785 D^2$, where D is the distance in kilometer (km) from the level to the staff station, and e is in meters. In normal levelling, sight length is less than 300 m, hence e will always be less than 0.007 m.

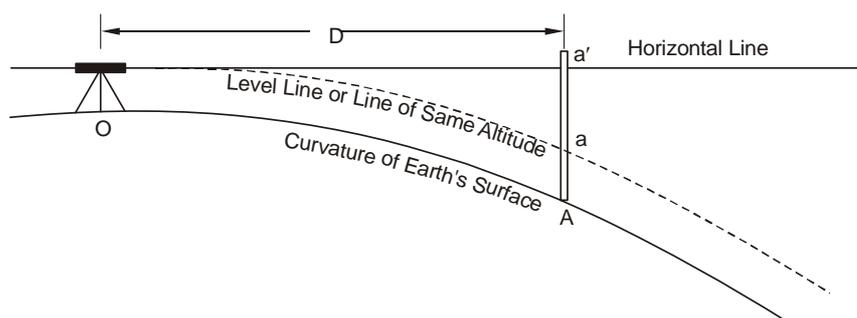
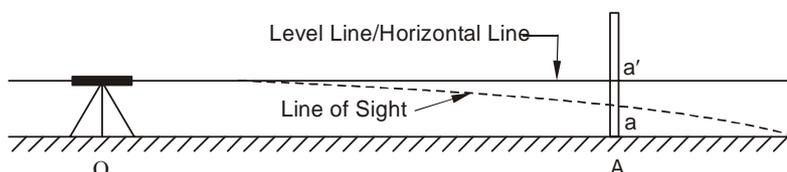


Figure 4.12 : Error Due to Curvature

Errors due to refractions are introduced due to refraction of light passing through layers of air of different densities. The bent light ray from staff to instrument will not remain horizontal (Figure 4.13) but will be curved introducing error aa' . The effect of refraction is not constant but varies with atmospheric conditions. However, on an average under normal



atmospheric conditions the correction for refraction will be aa' . The error, e_r (in meters) = $0.0112 D^2$ (i.e. roughly about 1/7 the correction due to curvature and opposite in sign).

The combined correction due to curvature and refraction would be

$$e_{co} = e_c - e_r = (0.0785 - 0.0112) D^2 = 0.0673 D^2$$

As the effect of curvature is to increase the staff reading so the correction for curvature is subtractive. The correction for refraction is additive to staff reading. Hence, the combined correction is subtractive to staff reading.

SAQ 2



Write short notes on the following

- (a) Profile leveling
- (b) Sources of errors in leveling
- (c) Reciprocal leveling

4.6 CONTOURING

Contour lines on maps are plotted to show the variation in the elevation of earth surface in plan for various engineering purposes. Contours are used in a variety of engineering works like location of roads, canals, water supply, water distribution, planning and designing of dams, reservoirs, aqueducts, transmission lines, estimating capacity of reservoirs etc.

4.6.1 Definitions

Contour

A contour is an imaginary line on the ground passing through points of equal elevation. It may also be defined as a line in which a level surface intersects the earth's surface. A contour line on a map represents a contour of particular elevation.

Contour Interval

The vertical distance between any two consecutive contours is called contour interval. Contour interval is kept constant for a contour plant. Contour interval depends upon the following factors :

- (a) Nature of the ground,
- (b) Scale of the map,
- (c) Purpose and extent of survey, and
- (d) Time and funds.

Nature of the Ground

Contour interval varies with the topography of the area. If the ground is steep, the contour interval will be large, whereas for flat grounds the contour interval will be small.

Scale of the Map

Contour interval is inversely proportional to the scale of the map. If the scale is large the contour interval should be small, whereas if the scale is small the contour interval should be large.

Purpose and Extent of Survey

Purpose and extent of survey affects the choice of contour interval, e.g. small contour interval is used for a survey intended for detailed design work and for accurate earthwork calculations. A large contour interval is used when the extent of survey is large, e.g. location surveys for communication lines, highways and railways.

Time and Funds

If the time and funds are short and limited the contour interval is kept large.

Horizontal Equivalent

The horizontal distance between any two consecutive contours is known as horizontal equivalent. For a given contour interval horizontal equivalent depends upon the steepness of the ground.

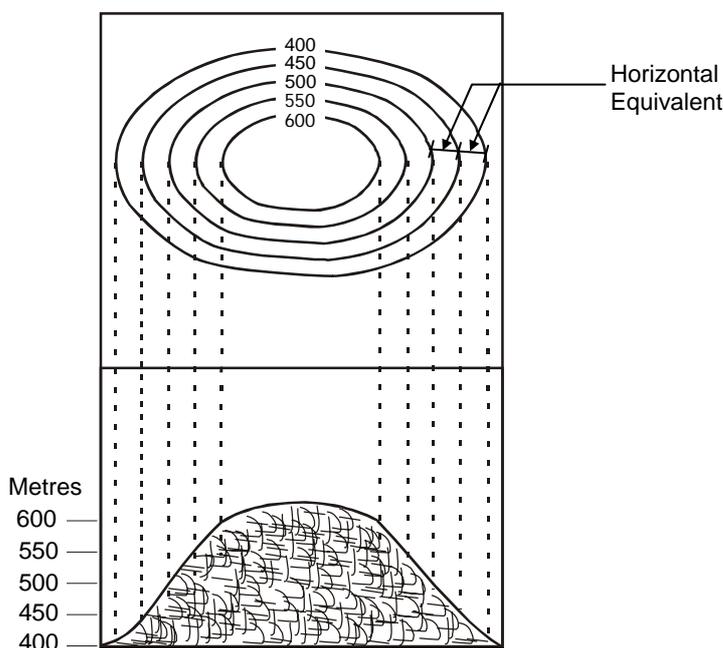


Figure 4.14: Contour Representation of a Plateau

4.6.2 Characteristics of Contour Lines

Characteristics of contour lines are helpful in plotting and interpretation of various features in the map. These characteristics are as follows :

- (a) Contour line is a line joining points of same elevation, hence all points of contour lines have same elevation. The elevation of a contour is written close to the contour.

- (b) Two contour lines of different elevations cannot intersect each other except in case of an overhanging cliff or a cave (Figure 4.15).

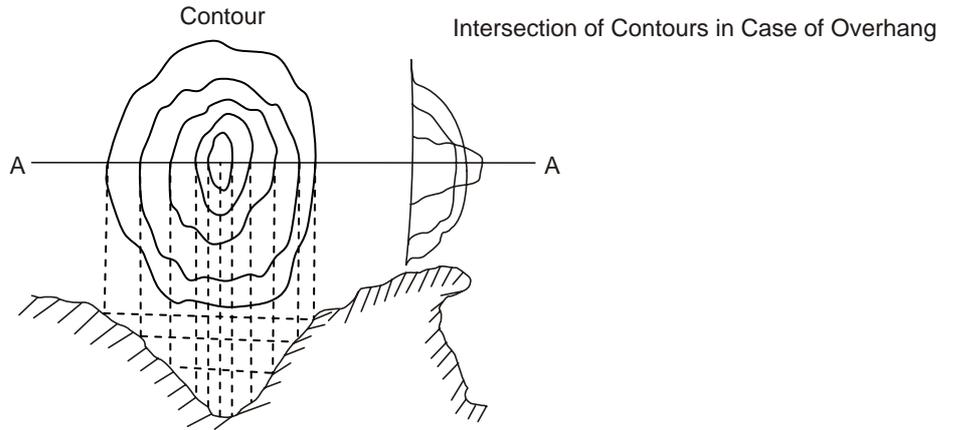


Figure 4.15 : Section of Ground Surface at A-A

- (c) In case of a vertical cliff contour lines of different elevations can join to form one single line.
- (d) Horizontal equivalent of contours indicates the topography of the area. The uniformly spaced contour lines indicate a uniform slope, while straight and equally spaced contour line indicate a plane surface. Contour lines closed together indicate steep slope, while a gentle slope is indicated when contour lines are far apart.

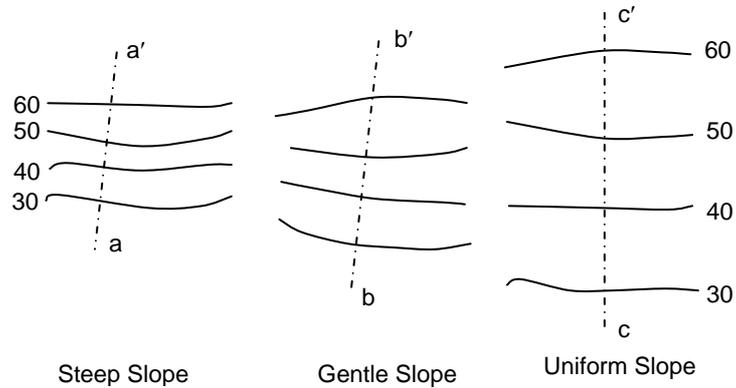
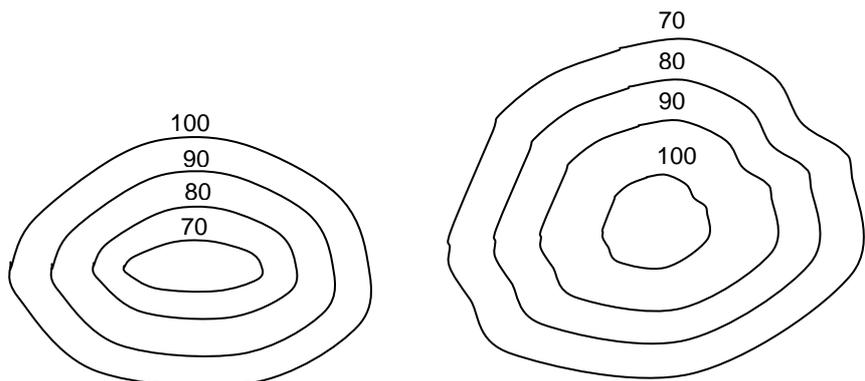


Figure 4.16: Topography of Area Represented by Variation of Horizontal Equivalent

- (e) A contour line cannot end anywhere and must close upon itself, though not necessarily within the limits of the map.
- (f) A set of close contours with higher figures outside and lower figures inside indicate a depression or lake, whereas a set of close contours with higher figures inside and lower figures outside indicate a hillock.



(a) Depression

(b) Hillock

Figure 4.17 : Set of Contour Showing Depression and Hillock

- (g) Contour lines cross a water shed (or ridge line) and a valley line at right angles. In case of ridge line, they form curves of U-shape across it with concave side of the curve towards higher ground, whereas in case of valley line, they form sharp curves of V-shape across it with convex side of curve towards higher ground.

4.6.3 Methods of Locating Contours

Methods of locating contours may be classified as (a) direct, and (b) indirect.

Direct Method

In this method, the contour to be plotted is actually located on the ground with the help of a level or hand level by marking various points on the contour. These points are surveyed and plotted to draw the contours through them on the plan. Though the method is slow and tedious but it is most accurate and is used for contouring small areas with great accuracy.

In contouring, field work consists of horizontal and vertical control. For a small area, horizontal control can be performed by a chain or tape, while for a large area compass, theodolite or a plane table can be employed. For vertical contour, a level and staff or a hand level may be used.

Vertical Control by Level and Staff

A series of points having same elevation are located on the ground in this method. An instrument station on the ground is selected so that it commands a view of most of the areas to be surveyed. Height of the instrument can be fixed sighting a nearest benchmark. Staff reading is calculated for a particular contour elevation. The staff man is directed to move left or right along the expected contour until the required reading is observed. A series of points having same elevation as shown by the same staff reading are plotted and joined to get a smooth curve.

Vertical Control by Hand Level

The same principle as used in level and staff method is employed in this method also. This method is very rapid in comparison to the former method. A hand level or an abney may be used to get an indication of the horizontal line from the eye of the observer. A level staff or a pole having zero mark at the height of the observer's eye which is graduated up and down from this point is used in this method. The man with the instrument stands over the benchmark and the staff man is moved to a point on the contour to be plotted. As soon as the man with instrument observes the required staff reading for a particular contour he instructs the staff man to stop and locates the position of the point.

Indirect Method

Indirect methods are quicker, cheaper and less laborious than direct method. In this method, a series of guide points are selected along a system of straight lines and their elevations are determined. These points are then plotted and contours are drawn by interpolation. The guide points generally are not the points on the contours to be located except in case of a

coincidence. For plotting of contours, the interpolation is done with the assumption that the slope between any two adjacent guide points is uniform. Some of the indirect methods of locating ground points are given below.

Methods of Squares

This method is very suitable when the area to be surveyed is small. This method is also called coordinate method of locating contours. The area to be surveyed is divided into a number of squares forming a grid. The side of a square may vary from 5 to 20 m depending upon the nature of the contour and contour interval. The elevations of the corner of squares are then determined by using a level and a staff. The levels are then interpolated and contour lines are drawn. Sometimes rectangles may also be used in place of squares.

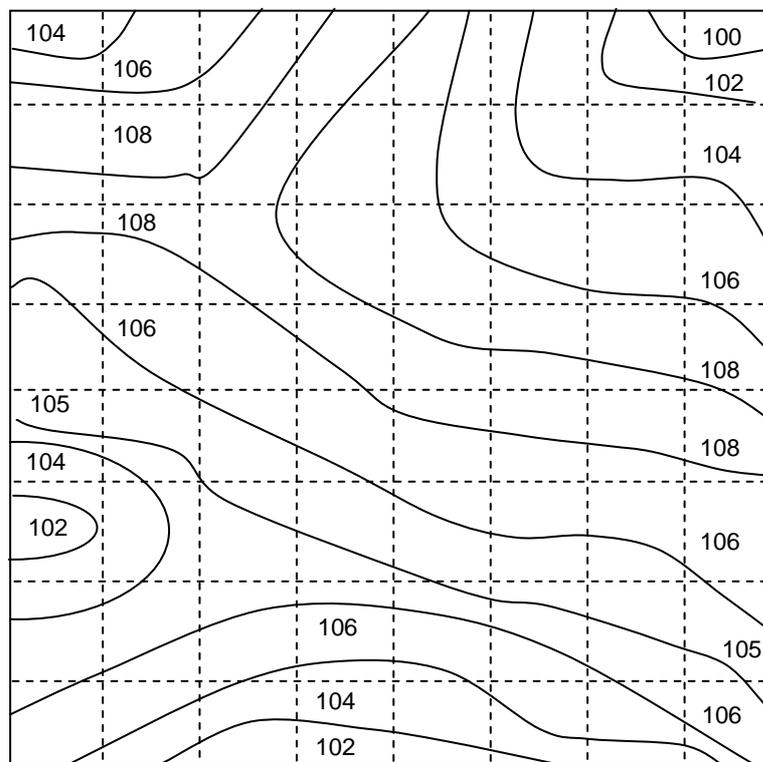


Figure 4.18 : Method of Squares for Locating Contours

Methods of Cross-sections

This method is generally used in route surveys. Cross-sections are run transverse to the centre line of a canal, road and railway etc. The spacing of cross-sections basically depend on the nature of terrain and the contour interval. The reduced level of various points along the section line are plotted on the plan and the contours are then drawn by interpolation.

Tacheometric Method

This method is suitable for hilly areas. In this method, a number of lines are set out radiating at a given angular interval from different traverse stations. The representative points on these lines are located in the field by observing vertical angles and the staff reading of the stadia wires of a tacheometer. The elevations and the distances of

these points are calculated and plotted and then contour lines are drawn by interpolation.

4.6.4 Interpolation of Contours

Interpolation of contours is the process of spacing the contours proportionally between the plotted ground points. Contours may be interpolated by the following methods.

By Estimation

This is a rough method and is used on a small scale maps. In this method, position of contour points between ground points are estimated and the contour lines are drawn through these points.

By Arithmetic Calculations

This is very accurate method but time consuming also. Position of contours points between guide points are located by arithmetic calculations. For example, *A* and *B* are two ground points having their elevations as 53.65 m, and 56.85 m respectively. The distance between these points is 15 m and let the contour interval is 1 m. Let between *A* and *B* the contours of 54 m, 55 m and 56 m are to be located. The contours can be located as follows :

Difference of level between *A* and *B* = $56.85 - 53.65 = 3.20$ m.

Difference of level between *A* and the 54 m contour point
= $54 - 53.65 = 0.35$ m.

Hence, distance of 54 m contour point from *A* = $(0.35/3.2) \times 15 = 1.64$ m.

Similarly, the distances of 55 m and 56 m contour points from *A* can be calculated as 6.33 m and 11.01 m, respectively. These distances can be plotted to the scale on the map.

By Graphical Method

This method is rapid and convenient when high accuracy is required and interpolation work is too much. In this method, a tracing paper/cloth is used to draw parallel lines at some fixed intervals, say 0.5 m, at equal intervals and every 10th line is made thicker. Let *A* and *B* are two points of elevation 57.5 m and 68.5 m, respectively. Suppose it is required to interpolate 5 m contours between *A* and *B*. Assume that bottom or zero line represents an elevation of 55 m and the successive thicker line represents 60 m, 65 m, 70 m etc. Place the tracing paper so that the point *A* is on the 5th line. Now, turn the tracing paper until the point *B* is on the 27th line from the zero line (or on the 7th line from the second thicker line). The intersections of 1st and 2nd thicker line representing elevations 60 m and 65 m and the line *AB* will give the positions of the points on the 60 m and 65 m contours, respectively. These positions are pricked through on to the plan.

4.6.5 Applications

Some of the important uses of contour maps are as follows :

- (a) Intervisibility of two given points can be ascertained from the contour map.
- (b) Inspection of contour map can provide information regarding character of the tract of the country, e.g. whether it is flat, undulating or mountainous etc.

- (c) Contour maps can help in selection of most economical and suitable site for engineering works like reservoir, canal, sewer, road or railway.
- (d) Earthwork computation can be done with the help of the contour maps.
- (e) From a given contour plan the section along any given direction can be drawn to know the general shape of the ground or to use it for earthwork calculations.
- (f) Contour plan may be used to calculate the capacity of the reservoir.

SAQ 3



- (a) Write down the factors that affect the choice of contour interval.
- (b) Write the important characteristics of contour lines.
- (c) What is the difference between direct method and indirect method of locating contours?
- (d) What are the different methods of interpolation of contours?

Example 4.1

Following readings were taken during a levelling exercise. The instrument was shifted after 5th and 8th reading. Enter above readings in the regular fieldbook format and find *RLs* if the *RL* of the starting station is assumed to be 100.000.

2.432, 3.446, 3.013, 2.006, 0.847, 2.689, 2.784, 1.667, 0.974, 0.832 and 0.168.

Solution

Filled Page of Field Book is given here.

Station	Staff Reading			HI (Collimation Level)	RL	Remarks
	BS	IS	FS			
A	2.432	–	–	102.432	100.000	Starting Station
1	–	3.446	–	–	98.986	Instrument set at O ₁
2	–	3.013	–	–	99.419	
3	–	2.006	–	–	100.426	
B	2.689	–	0.847	104.274	101.585	Change Station B (IS O ₂)
4		2.784			101.490	
C	0.974	–	1.667	103.581	102.607	Change station C (IS O ₃)
5		0.832			102.749	

D			0.168		103.413	Last Station
Sum	6.095	–	2.682			

- (a) Number of backsights = Number of foresights = 3.
 (b) First entry on the page is backsight while the last entry is a foresight.
 (c) Height of instrument at station $O_1 = \text{RL of BM} + \text{BS at A}$
 $= 100.000 + 2.432 = 102.432 \text{ m.}$

$$\begin{aligned} \text{RL of intermediate station 1 (ISI)} &= HI_1 - \text{IS at 1} \\ &= 102.432 - 3.446 = 98.986 \text{ m} \end{aligned}$$

$$\text{RL of IS 2} = 102.432 - 3.013 = 99.419 \text{ m}$$

$$\text{RL of IS 3} = 102.432 - 2.006 = 100.426 \text{ m}$$

$$\text{RL of change station B} = 102.432 - 0.847 = 101.585 \text{ m}$$

- (d) HI at station $O_2 = \text{RL of B} + \text{BS at B}$
 $= 101.585 + 2.689 = 104.274 \text{ m}$

$$\text{RL of IS 4} = 104.274 - 2.784 = 101.490 \text{ m}$$

$$\text{RL of change station C} = 104.274 - 1.667 = 102.607 \text{ m}$$

- (e) HI of change station $O_3 = 102.607 + 0.974 = 103.581 \text{ m}$

$$\text{RL of IS 5} = 103.581 - 0.832 = 102.749 \text{ m}$$

$$\text{RL of last station D} = 103.581 - 0.168 = 103.413 \text{ m}$$

$$\text{Check } \sum BS - \sum FS = \text{Last RL} - \text{First RL}$$

$$\text{i.e. } 6.095 - 2.682 = 103.413 - 100.000$$

$$3.413 = 3.413.$$

Example 4.2

Solve Example 4.1 by rise and fall method.

Solution

First 4 columns will be the same as in Example 4.1.

Station	Levels			Height		RL	Remarks
	BS	IS	FS	Rise	Fall		
A	2.432	–	–	–	–	100.000	First Reading
1	–	3.446	–	–	1.014	98.986	
2	–	3.013	–	0.433	–	99.419	
3	–	2.006	–	1.007	–	100.426	
B	2.689	–	0.847	1.159	–	101.585	Change Station B
4	–	2.784	–	–	0.095	101.490	
C	0.974	–	1.667	1.117	–	102.607	Change Station C
5	–	0.832	–	0.142	–	102.749	
D	–	–	0.168	0.664	–	103.413	Last

							Reading
Sum Σ	6.095		2.682	4.522	1.109	-	-

For Point 1, Fall will be equal to $3.446 - 2.432 = 1.014$ m

$$RL \text{ of Point 1} = 100.000 - 1.014 = 98.986$$

Fall of Point 2 = $(3.013 - 3.446) = (-0.433)$ Rise

$$RL \text{ of point 2} = 98.986 - (-0.433) = 99.419$$

Fall of point 3 = $2.006 - 3.013 = (-1.017)$ Rise

$$RL \text{ of point 3} = 99.419 - (-1.017) = 100.426$$

Fall of Point B = $0.847 - 2.006 = (-1.159)$ Rise

$$RL \text{ of Point B} = 100.426 - (-1.159) = 101.585$$

Instrument Station shifted from O_1 to O_2

Fall of point 4 = $2.784 - 2.689 = 0.095$

$$RL \text{ of Point 4} = 101.585 - 0.095 = 101.490$$

Fall of point C = $1.667 - 2.784 = (-1.117)$ Rise

$$RL \text{ of point C} = 101.490 - (-1.117) = 102.607$$

Fall of Point 5 = $0.832 - 0.974 = (-0.142)$ rise

$$RL \text{ of point 5} = 102.607 - (-0.142) = 102.749$$

Fall of Point D = $0.168 - 0.832 = -0.664$

$$RL \text{ of Point D} = 102.749 - (-0.664) = 103.413 \text{ Answer}$$

$$\text{Check } \Sigma BS - \Sigma FS = 6.095 - 2.682 = 3.413$$

$$\text{Sum of Rise} - \text{Sum of Fall} = 4.522 - 1.109 = 3.413$$

$$\text{Last } RL - \text{First } RL = 103.413 - 100.000 = 3.413$$

Example 4.3

A dumpy level was set up with telescope vertical over the Peg driven at station O_1 such that top of O_1 is 1.745 m from centre of telescope. The foresight taken on peg O_2 was 1.123 m. The level was then moved and set at station O_2 . The height of centre of telescope from top of O_2 is 0.824 m and the reading on staff held at O_1 is 1.438 m. If the RL of O_1 is given as 104.646 m, find the true RL of O_2 .

Solution

(a) Level at O_1 : Staff reading at $O_1 = 1.745$ m

$$\text{Staff reading at } O_2 = 1.123 \text{ m}$$

$$\text{Level difference between } O_1 \text{ and } O_2 = 0.622 \text{ m}$$

(b) Level at O_2 : Staff Reading at $O_2 = 0.824$ m

$$\text{Staff reading at } O_1 = 1.438 \text{ m}$$

$$\text{Level difference between } O_1 \text{ and } O_2 = 0.614$$

There is an error in collimation line. It is not parallel to bubble tube axis.

Actual level difference between O_1 and O_2 will be the average of the above two level differences = $(0.622 + 0.614)/2 = 0.618$ m

RL of $O_1 = 104.646$

Add true rise = 0.618 m

RL of $O_2 = 104.646 + 0.618 = 105.264$ m.

SAQ 4



- (a) RL of a floor in an industrial complex is 64.500 m. Staff reading on the floor is 1.715 and the staff reading when it is held inverted with bottom touching the ceiling of the room is 2.970. Find the height of the ceiling above the floor.
- (b) The following consecutive readings were taken during a levelling exercise on a continuously sloping ground at each chain length (30 m) 0.780, 1.535, 1.955, 2.430, 2.985, 3.480, 1.155, 1.960, 2.365, 3.640, 0.935, 1.045, 1.630, and 2.545.

Reduced level of first point is 150.000 m. Enter the above readings in a standard field book page and calculate the RL of each point. Also, the gradient of the line joining the first and the last point.

SAQ 5



- (a) Find the combined correction for curvature and refraction for distance of 3400 m and 12900 m.
- (b) In levelling between two points A and B on opposite banks of a river, the level was set up near A , and the staff reading on A and B were 1.285 and 2.860 m respectively. The level was then moved and set up near B and the respective readings on A and B were 0.860 and 2.220. Find the true difference of level between A and B .

4.6 SUMMARY

In this unit, you have studied about one of the important aspects of surveying, i.e. levelling. The objective of levelling is to determine the relative height of points of interest on the earth's surface. Levelling involves measurement of distances in a vertical plane, which is done with the help of a level and a levelling staff. There are different types of level and levelling staff, which are used for levelling work. Step by step procedure of levelling exercise is explained. Different methods of levelling have been discussed along with methods of reduction of levels.

Contouring and characteristics of contours are explained. Different types of errors in levelling, their cause and remedial measures are also explained.

4.7 ANSWERS TO SAQs

SAQ 4

- (a) 4.685 m.

SAQ 5

- (a) 0.779 m, 0.112 m.
(b) 1.468 m.